Tutorial No. 1

Data representation and encoding

Exercise 1:

1) Convert the following to decimal numbers.

   $(10011)_2$,$(00000001)_{2,}$ $(1111)_{2,}$ $(62)_8$, $(777)_8$, $(2F3A)_{16,}$

2) Convert the decimal numbers 8, 16, 64, and 128 into binary (base 2), octal (base 8), and hexadecimal (base 16).
3) Convert the binary number $(1001\ 0010\ 0100\ 1001)_2$, into octal (base 8), and hexadecimal (base 16).

Exercise 2 :

1) Convert the hexadecimal number F03B to binary (base 2) and octal (base 8).
2) How many non-negative integers can be represented in binary using 2 bytes? What is the maximum value?

Exercise 3 :

Represent the following signed decimal numbers in three different representations using an 8-bit encoding. "sign and magnitude", "one's complement," and "two's complement"

+9, -25, +127, -127, +130, -130.

Exercise 4 :

Our memory word size is 16 bits. Represent the following numbers using 2's complement notation:

1°/ +8 3°/ +16 5°/+129

2°/ - 8 4°/ -16 6°/-129

Exercise 5:

Let's examine the signed integer number N1, with N1 = -70.

1°) Give the binary value of the integer N1 on 8bits according to the 3 representations, “sign and magnitude", "one's complement," and "two's complement"

2°) Find the 8-bit decimal and binary representation of the signed integer N2 for each of the following scenarios:

A°/ $(N2)_2=(N1)_2+(0000\ 0001)_2$ where numbers are represented in 8-bit sign and magnitude (SM).

B°/ $(N2)_2=(N1)_2+(0111\ 1111)_2$ where numbers are represented in 8-bit one's complement (1CP).

C°/ N1-N2=5 where numbers are represented in 8-bit tow’s complement (2CP)

Exercices Supplémentaires

Exercise 1 :

1) Convert $(13CA)_{16}$, $(4002)_5$ et $(11110101)_2$ to decimal numbers.
2) Encode the decimal number 1176 successively in base 2, 8 and 16.

Exercise 2 :

1) Convert the following binary numbers to to decimal :

a°/$\mathbf{10000}_2$ **b°**/ 10001_2 **c°**/ 100000_2 **d°**/ 101010_2

2) Give the decimal number 2023 if it is encoded in a base of 8 or a base of 16.

Exercice 3 :

Encode the following decimal numbers on 8 bits, in sign and magnitude representation, in one's complement and tow's complement:

a°/ ±122 b°/ ±64 c°/ ±37

Exercice 4 :

Using 16 bits in 2's complement, give the binary, octal and hexadecimal values of the following decimal numbers: **a°/ -100 b°/ -255 c°/ -1023 d°/ -1027**

Exercice 6:

Give the decimal value of numbers coded in 2's complement on 16 bits:

a°/ $\mathbf{(77440)_8}$ **b°/** $\mathbf{(A000)_{16}}$ **c°/** $\mathbf{(F00F)_{16}}$ **d°/** $\mathbf{(7F01)_{16}}$

Exercice 7 :

Two signed integer numbers X and Y are encoded in 8-bit binary, where X=63.

1°) Find the decimal value of Y, Considering that both numbers are represented in 2's complement 8 bits notation and that X = -Y.

2°) Find the decimal value of Y, Considering that both numbers are represented in 1's complement 8 bits notation and that X+Y=-19.

3°) Donnée Find the decimal value of Y, Considering that both numbers are represented in 2's complement 8 bits notation and that X+Y=-19.

Solutions

Exercice 1 :

1) $(10011)_2$= $1*2^0+1*2^1+0+0+1*2^4=1+2+16=19$.

$(00000001)_2=1*2^0+0+0+0+0+0+0=1$

$(1111)_2=1*2^0+1*2^1+1*2^2+1*2^3=15$ ou $=2^4-1=16-1=15$

$(62)_8=2*8^0+6*8^1=2+48=50$.

$(777)_8$ (7 n'existe pas dans la base 8)

$(2F3A)_{16}=10*16^0+3*16^1+15*16^2+2*16^3=10+48+3840+8162=12090$

2) $8=(1000)_2$ $8=(10)_8$ $8=(8)_{16}$ Division by base
$16=(10000)_2$ $16=(20)_8$ $16=(10)_{16}$
$64=(100\ 0000)_2$ $64=(100)_8$ $64=(40)_{16}$

3) $(001\ 001\ 001\ 001\ 001\ 001)_2=(111111)_8$.
1 1 1 1 1 1

$(1001\ 0010\ 0100\ 1001)_2=(9249)_{16}$.
9 2 4 9

Exercise 2 :

1) $(F03B)_{16}=(1111\ 0000\ 0011\ 1011)_2$.
$(F03B)_{16}=(170073)_8$.
2) 2 octet =16 bits
We can code 2^{16} unsigned integers.
The maximum value is $2^{16}-1$.
In the standard situation, we have the ability to encode using n bits, 2n unsigned integer.
The largest value is 2n-1.

Exercise 3:

+9 = (+) → 0000 1001 SM,1cp et 2cp
-25 = (-) → 1001 1001 SM
0001 1001 SM de +25
1110 0110 1cp
+ 1
= 1110 0111 2cp

+205 = (+) → 011001101 SM,1cp et 2cp

-198 = (-) → 1 1100 0110 SM
0 1100 0110
1 0011 1001 1cp
+ 1
1 0011 1010 2cp

+127 = (+) → 0111 1111 SM,1cp et 2cp
-127 = (-) → 1111 1111 SM
0111 1111 SM de +127
1000 0000 1cp
+ 1
= 1000 0001 2cp

+130 = (+) → 1000 0010 SM, 1cp et 2cp
Capacity overflow
-130 = (-) → 1000 0010 SM

Exercise 4 :

1°/ +8 (+)→ 8=0000 0000 0000 1000 SM, 1cp et 2cp

2°/ -8 (-) → 0000 0000 0000 1000 SM de (+8)

1111 1111 1111 0111 1CP

+ 1

= 1111 1111 1111 1000 2CP

Exercice 5 (EF1 2014-2015):

N1=-70.

1°) SM : 1100 0110 1CP : 1011 1001 2CP : 1011 1010

2°) A°/ $(N2)_2=(N1)_2+(0000\ 0001)_2$

⇒ 1100 0110

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0000 0001

N2=$(1100\ 0111)_2$ sur 8bits SM→N2<0 donc N2=-71

B°/ $(N2)_2=(N1)_2+(0111\ 1111)_2$

⇒ 1011 1001

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0111 1111

N2=$(1\ 0011\ 1000)_2$ sur 8bits 1cp→N2>0 donc N2=+56

C°/ method 1 :

N1-N2=5 ⇒ N2=N1-5 en 2CP

⇒ 1011 1010

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0000 0101

N2=$(1011\ 0101)_2$ N2<0 2cp

$(0100\ 1010)_2$ +1=$(0100\ 1011)_2$

N2= -75

method 2 :

N1-N2=5 ⇒ N2=N1+ (-5) en 2CP

⇒ 1011 1010

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1111 1011 -5 en 2cp

N2=$(1\ 1011\ 0101)_2$ N2<0 2cp

$(0100\ 1010)_2$ +1=$(0100\ 1011)_2$

N2= -75