Chapter 1 :

Data representation and encoding

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Information Encoding Definition

➢ A computer processes **data**.

it must be able to encode and display this data, , which can take many forms:

- Of various types
  - Numbers
  - Characters
  - Other (analog signals, images...)
- ❖ Of Various sizes
  - Fixed Size of X digits : Phone number, students number,…
  - Variable Size : name, adress, SMS, E-mail text, video movie...

Information Encoding Definition (2)

- Decimal (base-10) is the numbering system most commonly used in everyday life.

- It uses ten symbols (digit) (0 to 9) to represent numbers.

- Each digit's position represent a power of 10.

- Each digit has a specific place value based on its position from the right.

- The rightmost digit is the "ones" place, the next digit to the left is the "tens" place, the next is the "hundreds"...

Exemple: soit N= 2348

The Number Bases

- Decimal (Base 10) : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- Binary (Base 2) : 0, 1
- Octal (Base 8) : 0, 1, 2, 3, 4, 5, 6, 7 .
- Hexadecimal (Base 16): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

N.B.: digit in a binary number = bit (binary digit)

The Number Bases (2)

Table with the summary

of the first 16 numbers

of the bases 10,2,8,16

Décimal	Binaire	Octal	Hexadécimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Encoding unsigned number(natural)

Converting a decimal number to another base (base B)

- The process of converting a decimal number into a different base, denoted as "base B" :
  - ✓ Divide this number by **B**
  - ✓ Replace the original number with the result of the division and return any remainder.
  - ✓ Continue these steps until the original number becomes 0.
  - ✓ Read the recorded remainders, from right to left, to form the representation in base B.

Encoding unsigned number(natural) (2)

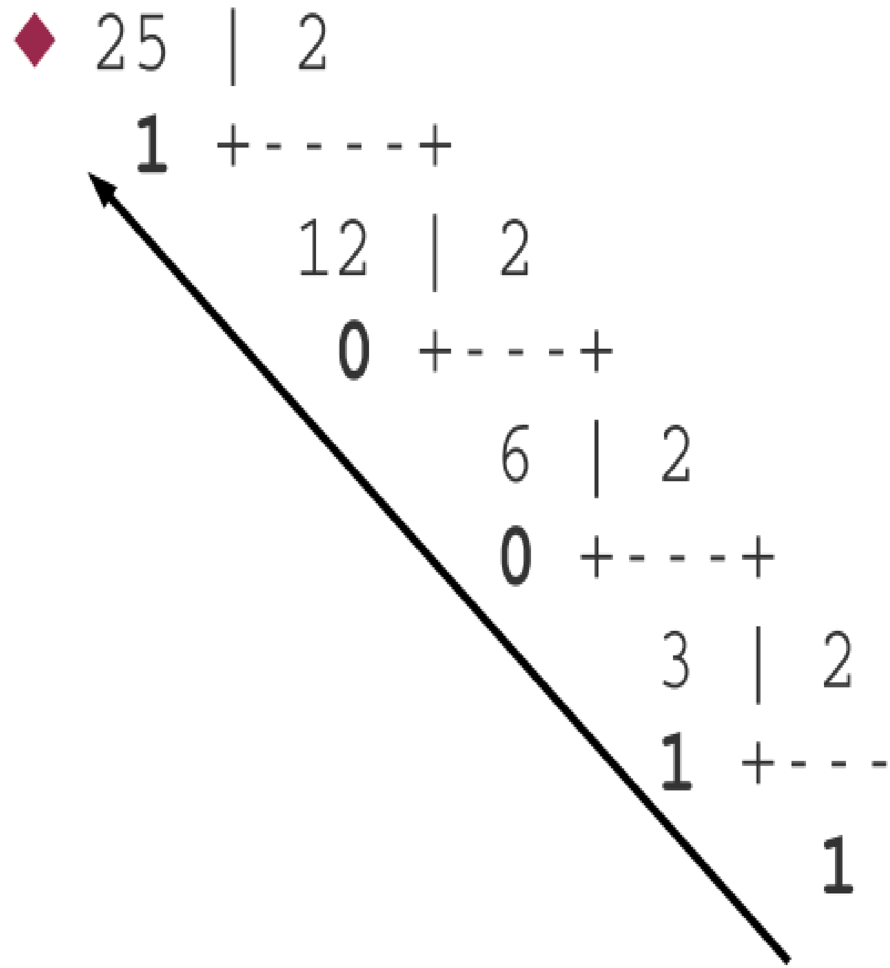
<u>Exemple 1: N=25</u>

convert to binary (B=2)

25/2=12 → remainder 1
12/2=6 → remainder 0
6/3=2 → remainder 0
3/2=1 → remainder 1
1/2=0 → remainder 1 **stop**

Result:

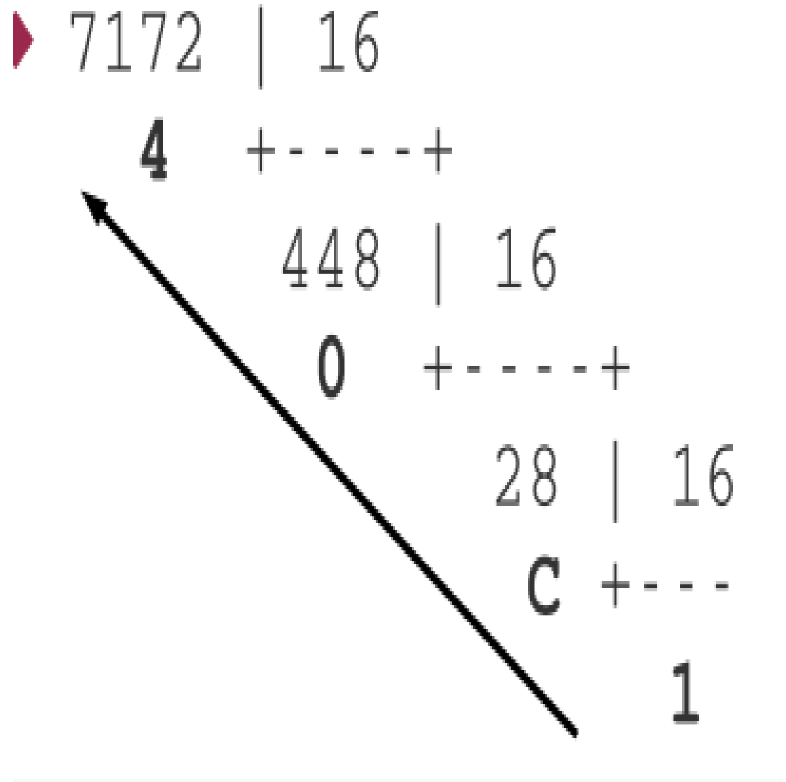
$N=25=(11001)_2$



Encoding unsigned number(natural) (3)

<u>Exemple 2: N=7172</u>

convert to Hexadecimal (B=16)



7172/16=448 → remainder 4
448/16=28 → remainder 0
28/16=1 → remainder 12
1/16=0 → remainder 1

stop

Result:

N=7172=$(1C04)_{16}$

Encoding unsigned number(natural) (4)

Converting from base B to Decimal

✓Start with the rightmost digit in the base B number.

✓Assign a positional value of 0 to the rightmost digit, and then increase the positional value by 1 for each digit to the left.

✓Multiply each digit by B raised to the power of its positional value.

✓Sum up the results of these multiplications for all digits.

✓The final sum is the decimal equivalent of the base B number.

$$a_n B^n + a_{n-1} B^{n-1} + \dots a_1 B + a_0$$ with $0 < a_i < B$

Encoding unsigned number(natural) (5)

Converting from base B to Decimal

Exemples:

- $(11001)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2 + 1$
  $= 16 + 8 + 1 = 25$
- $(1C04)_{16} = 1 \times 16^3 + 12 \times 16^2 + 0 \times 16 + 4$
  $= 4096 + 12 \times 256 + 0 + 4 = 7172$

Encoding unsigned number(natural) (6)

Converting between bases from base B to base B'

-First approach

Simply convert the number from base B to decimal and then from decimal to base B'.

Exemples: $(1100)_2 \longrightarrow (?)_8$

- $(1100)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0$
  $= 8 + 4 = (12)_{10}$
- 12|_8
  4|1|_8
  1| 0 ← je m'arrête

donc $(1100)_2 = (14)_8$

Encoding unsigned number(natural) (7)

Converting between bases from base B to base B’

–Second approach

from base 8,16 to base 2 or vice versa

To convert a number from base 8/16 to base 2, you can group each digit in base 8/16 as follow:

- ✓1 octal digit = one group of 3 binary digits
- ✓ 1 hexadécimal digit= one group of 4 binary digits

Encoding unsigned number(natural) (8)

Converting between bases from base B to base B’

–Second approach

Exemples:

$(1101)_2$ $(101)_2$ $(01101)_2$

$(31)_4$ $(11)_4$ $(31)_4$

$(1101)_2$ $(101)_2$ $(11101)_2$

$(15)_8$ $(5)_8$ $(35)_8$

$(1101)_2$ $(101)_2$ $(11101)_2$

$(D)_{16}$ $(5)_{16}$ $(1D)_{16}$

Binary arithmetic

- Addition binaire:

0+0=0

0+1=1

1+0=1

1+1=0 remainder 1

Example: addition of two unsigned 8-bit numbers

$$
\begin{array}{r}
(1000\ 1110)_2 \\
+\ (0001\ 0101)_2 \\
\hline
(1010\ 0011)_2
\end{array}
$$

Binary arithmetic (2)

- Soustraction binaire:

0 - 0=0

0 - 1=1 remainder 1

1 - 0=1

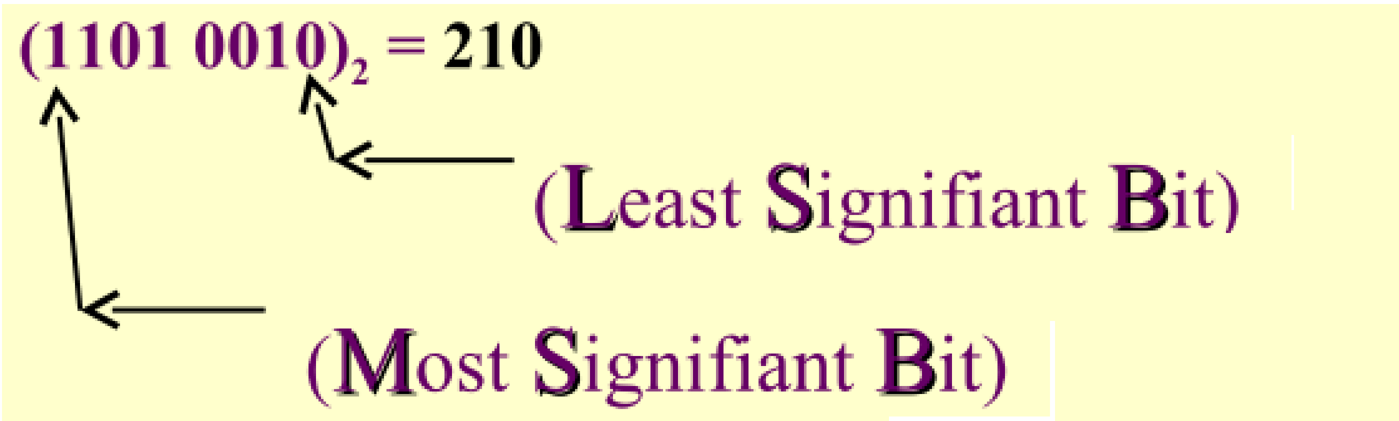
1 - 1=0

Example: subtraction of two unsigned 8-bit numbers

$$
\begin{array}{rr}
 & (1000\ 1110)_2 \\
- & (0001\ 0101)_2 \\
\hline
 & (0111\ 1001)_2
\end{array}
$$

Encoding signed numbers (integers)

➢ **MSB** and **LSB**



➢ When dealing with a signed number, the Most Significant Bit (MSB) serves as the sign indicator.

(1: negative and 0: positive).

Encoding signed numbers (integers) (2)

Three approaches exist for representing signed integers in binary base

- **Signed Magnitude Method (SM)**
- **One's complement (1cp)**
- **Two's complement (2cp)**

Encoding signed numbers (integers) (3)

➢Signed Magnitude Method (SM)

The number is divided into two parts:

Sign bit and magnitude, Sign bit is 1 for negative number and 0 for positive number.

Magnitude of number is represented with the binary form of the number.

Exemple: for 4 bits

$+7 = (\mathbf{0}111)_2$ because most significant bit (MSB) set to 0

$-7 = (\mathbf{1}111)_2$ = because least significant bit (LSB) set to 0

Disadvantage: always 2 ways to encoding the number 0

Encoding signed numbers (integers) (4)

➢ 1’s complement (1CP)

transforming the 0 bit to 1 and the 1 bit to 0

positive number : the same way as the unsigned number or Signed Magnitude Method.(SM)

negative number : complement of its positive opposite.

Exemple: encoding to one Byte

$+7 = (00000111)_2$ This is SM

1’s complement (1CP) : -7= (11111000)2

Disadvantage : Gives wrong results in arithmetic operations

Encoding signed numbers (integers) (5)

- **2’s complement (2CP)**

- is 1, added to the 1’s complement of the binary number.
- Positive number : the same way as the unsigned number or Signed Magnitude Method.(SM)

Exemple: with 4 bits
$+6 = (0110)_2$ This is SM
-6:
Complément à 1 : 1001
Complément à 2 : 1001 + 1 → $-6= (1010)_2$

Le codage ASCII

To store and transmit text, the computer must also use alphanumeric characters.
To code these characters, we associate a binary code with each one, This is called ASCII encoding (American Standard Code for Information Interchange).

Exemple:

- ✓Character A is code 65 or 01000001 in binary.
- ✓Character a is code 97 or 01100001 in binary.
- ✓Character f is: 102
- ✓the question mark? : 63
- ✓Number 2 : 50

Activity 1:

The numbers N, M and P 8-bit signed integer: N =+10.

1) Give the 8-bit binary value of N, in three representations: "signed and magnitude", "1's complement," and "2's complement".
2)

A) Give the binary and decimal value of the signed integer **P**=N-M, with M=$(0000\ 1100)_2$ and all numbers are represented as 8-bit and magnitude representation (SM).

B) Give the binary and decimal value of the signed integer **P** =N-M, with M=$(0000\ 1100)_2$, and all numbers are represented as 8-bit and 1's complement representation(1CP).

C) Give the binary and decimal value of the signed integer **P** =N-M, with M=$(0000\ 1100)_2$, and all numbers are represented as 8-bit and 2's complement representation(2CP). (***operations must be performed in binary).***

Activity for chapter No.1:

N=+10

1) $|N| = (1010)_2$

SM sur 8bits(N) ➔ N=$(0000\ 1010)_2$

1CP sur 8bits (N) ➔ N=$(0000\ 1010)_2$

2CP sur 8bits (N) ➔ N=$(0000\ 1010)_2$

2) M= $(0000\ 1100)_2$.

a) SM

Calcule de P en SM 0000 1010 SM(N)

\-

0000 1100 SM (M)

P=$(1111\ 1110)_2$ **SM (P<0)**

|M| is $(0111\ 1110)_2$

$M=0.2^0+ 1.2^1+1.2^2+1.2^3+1.2^4+1.2^5+1.2^6=2+4+8+16+32+64$

➔M= - 126

b) 1CP 0000 1010 1 CP(N)

\-

0000 1100 1CP(M)

P=$(1111\ 1110)_2$ 1 **CP (P<0)**

|M| est $(0000\ 0001)_2$

M=0.2^0 ➔**M= - 1**

c) 2CP 0000 1010 2CP (N)

-

0000 1100 2CP (M)

P=$(1111\ 1110)_2$ 2 **CP (P<0)**

$(1111\ 1110)_2$

-

1

1111 1101 CP

0000 0010 SM

|M| est $(0000\ 0010)_2$

M= 1.2^1 ➔**M= - 2**