Higher School in Applied Sciences
Preparatory Department
Course: Algebra 1
Academic Year: 2023/2024

Set of exercises 3

Exercise 1. Find the remainder of the division of the polynomial P by Q

$$P(X) = X^n + (X-1)^n + 1 \qquad Q(X) = X^2 - X$$

$$P(X) = (X sin\theta + cos\theta)^n \text{ with } n \in \mathbb{N}, \theta \in \mathbb{R} \qquad Q(X) = X^2 + 1$$

Exercise 2. For $n \in \mathbb{N}^*$ what is the order of multiplicity of 2 as the root of the polynomial:

$$P(X) = nX^{n+2} - (4n+1)X^{n+1} + 4(n+1)X^n - 4X^{n-1}$$

Exercise 3. Let $\alpha, \beta, \gamma, \delta \in \mathbb{N}$ and $n \in \mathbb{N}^*$ Show that Q divides P

$$P(X) = X^{4\alpha+3} + X^{4\beta+2} + X^{4\gamma+1} + X^{4\delta} \qquad Q(X) = X^3 + X^2 + X + 1$$

$$P(X) = nX^{n+1} - (n+1)X^n + 1 \qquad Q(X) = (X-1)^2$$

Exercise 4. Factor the following polynomials into $\mathbb{R}[X]$ then into $\mathbb{C}[X]$

1. $P_1(X) = X^3 + 4X^2 + 4X + 3$
2. $P_2(X) = (X^2 - X + 2)^2 + (X-2)^2$

To deduce $GCD(P_1, P_2)$ and $LCM(P_1, P_2)$ then $GCD(P_1, P_2, P_3)$ and $LCM(P_1, P_2, P_3)$

Exercise 5. Determine $GCD(P, Q)$ and $LCM(P, Q)$

$$P(X) = -2X^4 + 2X^3 + 2X - 2 \qquad Q(X) = 3X^3 + 9X^2 + 9X + 6$$

$$P(X) = X^n - 1 \qquad Q(X) = (X-1)^n \quad n \geq 1$$

Exercise 6. Decompose the following rational fractions in $\mathbb{R}(X)$

$$A = \frac{1}{X^3(X-2)^3} \qquad B = \frac{3}{(X^3-1)^2} \qquad C = \frac{X^3+1}{(X-1)^3} \qquad D = \frac{X^3}{X^4+X^2+1}$$

Exercise 7. Using Partial-fraction decomposition, calculate the following sums:

$$S_1 = \sum_{k=1}^{100} \frac{1}{k(k+1)} \qquad S_2 = \sum_{k=1}^{100} \frac{2k+1}{k^2(k+1)^2}$$