Sets

Definition 10. A set A is a collection of objects. If x is an object of A, we say that x is an element, or a member of A and we write $x \in A$. if x is not a member of A, we write $x \notin A$.

A set can be defined by the list of its elements or by setting conditions on its members.

Example 5. $A = \{0, 1, 2, 3\}$ Or $A = \{n \in \mathbb{N}\ n \leq 3\}$

A set that has no elements is called an empty set and denoted by $\emptyset$.

Definition 11. Suppose A is a set that has a finite number of elements $n \in \mathbb{N}$. Then n is said to be the cardinality of A and is denoted by $Card(A) = n$

Example 6. card$(A) = 4$ card$(\varnothing) = 0$

Subset

Definition 12. Let A and B be two non empty sets. We say that B is a subset of A (or that A is a superset of B) and we write $B \subseteq A$ if every element of B is also an element of A. That is

$$x \in B \Rightarrow x \in A$$

Definition 13. Two sets A and B are said to be equal if they have exactly the same elements. That is

$$A = B \Leftrightarrow (A \subseteq B) \text{ and } (B \subseteq A)$$

If $B \subseteq A$ and $B \neq A$, we say that B is a proper subset of A and we write $B \subsetneq A$ or more simply $B \subset A$

Definition 14. Let A be a nonempty set. The power set of A denoted by $\mathcal{P}(A)$ is the set whose elements are all the subsets of A. That is

$$\mathcal{P}(A) = \{X \mid X \subseteq A\}$$

Example 7. $A = \{0, 1\}$ $G = \{1\}$ $G \subset A$ and $G \in \mathcal{P}(A)$ $\mathcal{P}(A) = \{\varnothing, \{0\}, \{1\}, \{0, 1\}\}$

Operations on sets

Definition 15. The intersection of A and B, written $A \cap B$ and read "A intersect B," is the set of all elements that are in both A and B. That is,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

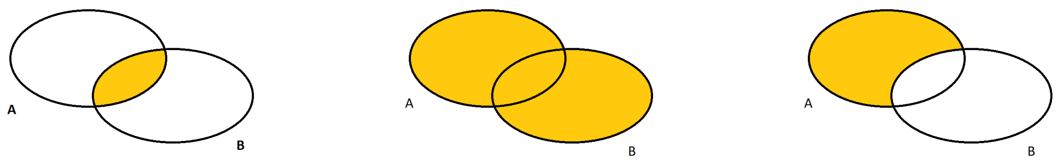
A and B are said to be disjoint if $A \cap B = \emptyset$.

Definition 16. The union of A and B, written $A \cup B$ and read "A union B," is the set of all elements that are in A or B. That is,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Definition 17. The set difference of A and B, written $A \setminus B$ and read "A minus B" is the set of all elements in A that are not in B. That is,

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$



(a) $A \cap B$ (b) $A \cup B$ (c) $A \setminus B$

Figure 1.1: Operations on sets

If $B \subset A$, then $A \setminus B$ is called the complement of B in A.
When A is understood from the context, the complement of of B in A is written B^c.

Example 8. Let $A =]-2, 5[$ and $B =]-4, 4]$, then
$A \cap B =]-2, 4]$ $\quad A \cup B =]-4, 5[$ $\quad A \setminus B =]4, 5[$ $\quad B \setminus A =]-4, -2]$

Theorem 4 (Morgan's Law)**.**

1. $(A \cap B)^c = A^c \cup B^c$
2. $(A \cup B)^c = A^c \cap B^c$

Definition 18. Let A and B be nonempty sets. The cartesian product of A and B denoted by $A \times B$ is the set defined by

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

We frequently read $A \times B$ as "A cross B." In the case where $A = B$, we will write $A \times A = A^2$.

Example 9. Let $A = \{a, b\}$ and $B = \{c, d\}$ then $(a, c) \in A \times B$ but is not an element of $B \times A$.